

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, AUGUST 2021

SECOND YEAR (BATCH 2019-22)

STATISTICS (General)

Date : 13/08/2021

Time : 11.00 am – 1.00 pm

Paper : IV

Full Marks : 75

Group - A

Answer **any six** questions of the following :

[6×5]

1. Show that the sample mean is an unbiased estimator for the population mean, where the population is normally distributed.
2. Derive the *MLE* of μ , when the observations $X_1, X_2, \dots, X_n$ are drawn from $N(\mu, \sigma^2)$.
3. What are sufficient conditions for an estimator to be consistent? Give an example of an estimator which is both unbiased and consistent. (3+2)
4. Explain, using the appropriate diagram, why the probability of type-I error increases when the probability of type -II error decreases.
5. When and how is the paired-t test used?
6. Suppose a random sample of size 10, drawn from a normal population, has mean 40 & s.d. 12. Find 99 % confidence limits for the population mean (Given $t_{0.005} = 3.25$ for 9 df.)
7. If $X_1, X_2, \dots, X_n$ is a random sample drawn from

$$f(x) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{x^2}{2\theta}}; -\infty < x < \infty$$

Show that $\frac{\sum X_i^2}{n}$ is an unbiased estimator of θ .

Group - B

Answer **any four** questions of the following :

[4×5]

8. Why is it necessary to attach weights to price relatives to get a proper index number?
Why are the weights used in Laspeyres' index number considered more meaningful than those considered in Paasche's index number? (2+3)
9. Let the price index numbers computed by Laspeyres' and Paasche's methods be denoted respectively by $L(p)$ and $P(p)$. Let similar quantities for quantity indices be defined and denoted as $L(q)$ and $P(q)$. Prove that if the values of any three of the above are known, the remaining one can be calculated.
10. What is the purpose of stratification in sampling? Discuss bias in sample survey? Why is the method of systematic sampling widely used in Practice? Illustrate your answer with some practical situations.
11. List the main components of a time series. With which component of time series would you associate each of the following? Why?
 - i) The rainfall that occurred in Calcutta for four days in February, 1981;
 - ii) A decline in ice cream sales during November to March;

iii) An era of prosperity;

iv) Increase in garment sales in October.

12. Suppose a simple random sample of size 2 is drawn from a finite population (y_1, y_2, y_3) . Corresponding to the three possible samples $S_1 = (y_1, y_2)$, $S_2 = (y_2, y_3)$ and $S_3 = (y_1, y_3)$ a linear estimator $l(S)$ for estimating the population mean is defined as
- $$l(S_1) = \frac{2}{3}y_1 + \frac{1}{2}y_2, \quad l(S_2) = \frac{1}{2}y_2 + \frac{1}{2}y_3, \quad l(S_3) = \frac{1}{3}y_1 + \frac{1}{2}y_3$$

Show that the estimator $l(S)$ is unbiased estimator of population mean and obtain its variance.

Group - C

Answer **any one** question from the following :

[1×10]

13. i) Ten individuals are selected at random from a normal population and the heights are found to be in inches : 63, 66, 63, 67, 68, 69, 70, 71, 68, 71. In the light of these data, carry out the test that population mean height is 66 inches. i.e. test

$$H_0 : \text{mean} = 66$$

$$H_1 : \text{mean} \neq 66$$

[Take level of significance as 0.05. You're provided with the information that $t_{0.025,9} = 2.262$]

- ii) Two random sample of sizes 9 and 8 give the sum of squares of deviations for the respective mean, equal to 180 and 100 respectively. Can they be regarded as drawn from the same normal populations. [Given $F_{8,7} = 6.84$ at $\alpha = 0.01$]

(5+5)

14. A die is thrown 120 times and the frequencies of different faces observed are as follows :

Face	1	2	3	4	5	6	Total
Observed Frequency	25	17	15	23	24	16	120

Test the hypothesis that the die is fair using a significance level 0.05.

[Given $P[\chi^2 > 11.1] = 0.05$ for 5df.]

Group - D

Answer **any one** question from the following :

[1×5]

15. A sample of size 20 is to be drawn from a population of size 300, using systematic sampling technique and find an unbiased estimator of the population mean and an unbiased estimator of its variance.

16. The following figures give the production of commodity of a country from 2001 to 2007:

Year	Production (in thousand metric tonnes)
2001	4.7
2002	5.9
2003	7.2
2004	7.9
2005	7.6
2006	7.0
2007	7.9

Obtain the trend values by a four year moving average method.

Answer **any one** question from the following :

[1×10]

17. A sequence of price indices was constructed with 2001 as base year and from 2004, the base year was changed to 2003. Using the data given below, splice the first series (with base 2001) to second series (with base 2003) and then calculate new indices by shifting the base to 2004.

Year	2001	2002	2003	2004	2005	2006
IN ₂₀₀₁	100	125	118.75	-	-	-
IN ₂₀₀₃	-	-	100	115	124.2	130.41

18. A simple random sample of 25 households was drawn from a city area containing 13745 households.

The number of persons per household in the sample were as follows:

5, 6, 3, 4, 2, 3, 3, 3, 4, 4, 3, 2, 7, 4, 7, 6, 4, 4, 1, 4, 3, 4, 5, 6, 3.

Estimate the total number of people in the area and also compute the variance of estimate.

TABLE VI
RANDOM SAMPLING NUMBERS *

4652	3819	8431	2150	2352	2472	0043	3488
9031	7617	1220	4129	7148	1943	4890	1749
2030	2327	7353	6007	9410	9179	2722	8445
0641	1489	0828	0385	8488	0422	7209	4950
8479	6062	5593	6322	9439	4996	1322	4918
9917	3490	5533	2577	4348	0971	2580	1943
6376	9899	9259	5117	1336	0146	0680	4052
7287	0983	3236	3252	0277	8001	6058	4501
0592	4912	3457	8773	5146	2519	3931	6794
6499	9118	3711	8838	0691	1425	7768	9544
0769	1109	7909	4528	8772	1876	2113	4781
8678	4873	2061	1835	0954	5026	2967	6560
0178	7794	6488	7364	4094	1649	2284	7753
3392	0963	6364	5762	0322	2592	3452	9002
0264	6009	1311	5873	5926	8597	9051	8995
4089	7732	8163	2798	1984	1292	0041	2500
9376	7365	7987	1937	2251	3411	6737	0367
3039	3780	2137	7641	4030	1604	2517	9211
8971	8653	1855	5285	5631	2649	6696	5475
0373	4153	5199	5765	2067	6627	3100	5716
9092	4773	0002	7000	7800	2292	2933	6125
2464	1038	3163	3569	7155	2029	2538	7080
3027	6215	3125	5856	9543	3660	0255	5544
5754	9247	1164	3283	1865	5274	5471	1346
4358	3716	6949	8502	1573	5763	5046	7135
7178	8324	8379	7365	4577	4864	0629	5100
5035	5939	3665	2160	6700	7249	1738	2721
3318	0220	3611	9887	4608	8664	2185	7290
9058	1735	7435	6822	6622	8286	8901	5534
7886	5182	7595	0305	4903	3306	8088	3899
3354	8454	7386	1333	5345	6565	3159	3991
3415	7671	0846	7100	1790	9449	6285	2525
3918	5872	7898	6125	2268	1898	0755	6034
6138	9045	6950	8843	6533	0917	6673	5721
3825	1704	2835	4677	4637	7329	3156	3291
1349	0417	9311	9787	1284	0769	8422	1077
4234	0248	7760	6504	2754	4044	0842	9080
6880	3201	7044	3657	5263	0374	7563	6599
0714	5008	5076	1134	5342	1608	5179	0967
3448	6421	3304	0583	1260	0662	7257	0766
5711	7343	7539	3684	9397	5335	4031	1486
2588	3301	0553	2427	3598	2580	7017	9176
8581	4253	7404	5264	5411	3431	3092	8573
8475	6322	3949	9675	6533	1133	8776	2216
0272	5624	8549	5552	7469	2799	2822	9620
7383	7795	7939	2652	4456	6993	2950	8573
5126	2089	7729	0945	3901	4445	7117	8186
2064	3760	0939	7319	5939	3432	2030	4752
9315	8185	7805	6294	7072	6491	4012	1016
6814	8752	3462	6001	3302	3895	7371	3432